Percentile-based Route PDFs from Segment PDFs
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It has long been generally accepted by experts in traffic flow analysis, including statisticians, that the variability of travel time for a segment cannot be determined by summing the variability in travel time of sections comprising the segment. For example, you cannot obtain the variance for a segment by first partitioning the segment into sections, determining the variance for each one, and adding the variances together. The same is true for the standard deviation and various other measures of reliability, such as the Planning Time Index (i.e. the Travel Time Index for the 95th percentile of the Travel Time Distribution)

The prohibition against adding segment variances (and similar calculations) is considered inviolable and poses many challenges, for example the inability to determine a measure of the travel time variability for a freeway section or an O-D pair based on summing the same measure of the variability of their parts.

A late-emergent finding from L02 is that one can create defensible distributions of travel times for a route (for specific regimes) by summing the percentile travel times across the segments that comprise the route (for the same regime). This is to say that *if* you have distributions of *individual vehicle travel times* for the *segments* that comprise a specific route (for a given regime), it is possible to create the distribution of *individual vehicle travel times* for the *route* (for that same regime) by summing the travel times for *specific percentiles* across the segments.

Consider the example shown in Table 1. Columns 2, 3, and 4 contain the travel times by percentile for a three-segment route along I-5 south of downtown Sacramento. The route starts at monitoring station #39 and proceeds past monitoring stations #9 and #10 and ends at monitoring station #11. So the first row in sub-table (a) shows the 5th percentile travel times on segments 39-9 (0.810 minutes), 9-10 (0.757 minutes), and 10-11 (0.778 minutes) under uncongested conditions. The fifth column shows the sum of these values (e.g., 0.810 + 0.757 + 0.778 = 4.450). The sixth column shows the percentile travel time that was actually observed for vehicles that traveled the entire route (e.g., the observed 5th percentile travel time for uncongested conditions was 4.500 minutes). The 7th column shows the percentage difference between the synthesized sum and the true, observed value. Sub-table (a) shows these values for uncongested conditions; sub-table (b) shows the results for low congestion; sub-table (c) for moderate congestion; and sub-table (d) for high congestion. Only when congestion on the facility is high can one see differences that are more than 1%. Moreover, not only are the percentage differences small, but adding the percentile values reproduces the multi-modal (two modal) distribution that was observed for the entire route, as shown in Figure 1.

This finding produces excitement that there may be a very simple way to estimate *individual vehicle travel time distributions* for *routes* on the basis of corresponding distributions for the *segments* that comprise the route (without being confined to utilize only the travel times of vehicles that actually

traversed the route. If it proves to be true generally, traffic management centers will be able to provide estimates of the density functions for user-requested routes by combining in this manner the density functions for the intervening segments.

If this idea could be tested more extensively, it would be of great benefit to the practicing traffic engineering community. While the idea upon which it is based is completely consistent with a premise upon which all traffic simulation models are based – that drivers consistently seek to achieve facility-dependent desired speeds as they make their trips through the network – it has only been demonstrated to be defensible on one multi-section stretch of I-5 in Sacramento. Finding that it is true far more generally would be a very significant contribution to knowledge.

Table 1: A Comparison of actual percentile travel times for a given route against values obtained by summing the travel times for the same percentile on the individual segments

	Segment Travel Times (Uncongs)					
Percentile	Seg 39-9	Seg 9-10	Seg 10--11	Sum	Route 39-11	% Diff
5%	0.810	0.757	0.778	4.450	4.500	1.11%
10%	0.829	0.773	0.814	4.566	4.600	0.74%
15%	0.843	0.788	0.833	4.650	4.667	0.36%
20%	0.856	0.803	0.852	4.733	4.733	0.00%
25%	0.866	0.818	0.861	4.792	4.800	0.17%
30%	0.875	0.834	0.870	4.850	4.867	0.35%
35%	0.884	0.848	0.889	4.916	4.917	0.02%
40%	0.896	0.864	0.908	4.992	4.983	0.18%
45%	0.903	0.871	0.926	5.041	5.050	0.18%
50%	0.917	0.886	0.944	5.125	5.117	0.16%
55%	0.926	0.902	0.972	5.200	5.200	0.00%
60%	0.940	0.916	0.991	5.283	5.300	0.32%
65%	0.958	0.939	1.000	5.383	5.433	0.92%
70%	0.977	0.955	1.019	5.484	5.550	1.19%
75%	0.995	0.977	1.037	5.591	5.625	0.60%
80%	1.012	1.000	1.056	5.692	5.700	0.14%
85%	1.028	1.015	1.074	5.784	5.783	0.02%
90%	1.046	1.038	1.092	5.892	5.875	0.29%
95%	1.074	1.075	1.130	6.067	6.017	0.83%

(a)

	Segment Travel Times (Low)					
Percentile	Seg 39-9	Seg 9-10	Seg 10--11	Sum	Route 39-11	% Diff
5%	0.806	0.743	0.778	4.417	4.467	1.12%
10%	0.824	0.773	0.797	4.534	4.567	0.72%
15%	0.838	0.788	0.814	4.617	4.650	0.71%
20%	0.849	0.803	0.833	4.691	4.717	0.55%
25%	0.861	0.818	0.852	4.767	4.767	0.00%
30%	0.870	0.818	0.861	4.808	4.833	0.52%
35%	0.880	0.834	0.870	4.867	4.883	0.33%
40%	0.889	0.848	0.889	4.933	4.942	0.18%
45%	0.898	0.864	0.908	5.000	5.000	0.00%
50%	0.908	0.879	0.926	5.067	5.067	0.00%
55%	0.921	0.894	0.944	5.150	5.150	0.00%
60%	0.933	0.909	0.972	5.233	5.233	0.00%
65%	0.949	0.925	0.991	5.326	5.350	0.45%
70%	0.968	0.939	1.009	5.424	5.467	0.79%
75%	0.986	0.962	1.019	5.525	5.567	0.75%
80%	1.002	0.985	1.037	5.624	5.650	0.46%
85%	1.019	1.000	1.064	5.725	5.733	0.14%
90%	1.037	1.030	1.092	5.849	5.817	0.55%
95%	1.060	1.061	1.130	6.001	5.950	0.86%

(b)

	Segment Travel Times (Mod)					
Percentile	Seg 39-9	Seg 9-10	Seg 10--11	Sum	Route 39-11	% Diff
5%	0.810	0.743	0.778	4.434	4.500	1.47%
10%	0.829	0.773	0.797	4.550	4.583	0.72%
15%	0.838	0.788	0.814	4.617	4.650	0.71%
20%	0.847	0.795	0.833	4.675	4.700	0.53%
25%	0.858	0.803	0.842	4.729	4.767	0.81%
30%	0.866	0.818	0.852	4.784	4.817	0.69%
35%	0.875	0.834	0.870	4.850	4.867	0.35%
40%	0.884	0.841	0.889	4.908	4.917	0.18%
45%	0.894	0.848	0.908	4.967	4.967	0.00%
50%	0.903	0.864	0.926	5.033	5.033	0.00%
55%	0.917	0.879	0.944	5.117	5.117	0.00%
60%	0.928	0.894	0.963	5.192	5.200	0.15%
65%	0.942	0.916	0.981	5.283	5.317	0.64%
70%	0.961	0.939	1.000	5.391	5.450	1.08%
75%	0.981	0.955	1.019	5.500	5.550	0.90%
80%	0.999	0.970	1.037	5.595	5.633	0.67%
85%	1.014	1.000	1.074	5.717	5.700	0.30%
90%	1.033	1.015	1.092	5.817	5.800	0.29%
95%	1.060	1.061	1.139	6.009	5.950	0.99%

(c)

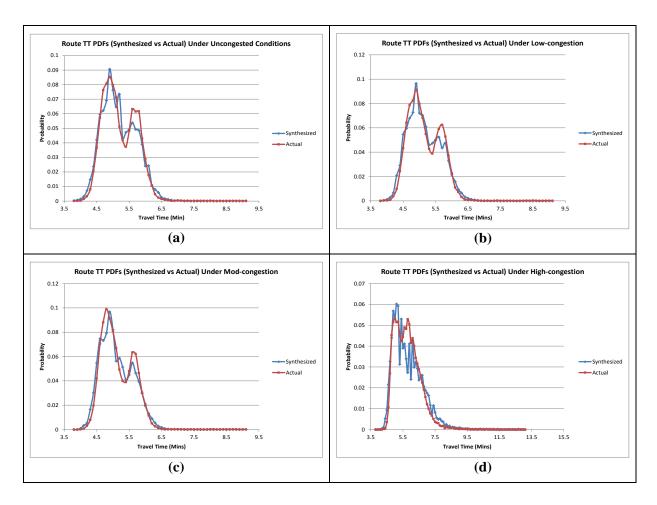|	Segment Travel Times (High)					
Percentile	Seg 39-9	Seg 9-10	Seg 10--11	Sum	Route 39-11	% Diff
5%	0.861	0.773	0.814	4.683	4.767	1.76%
10%	0.884	0.803	0.833	4.816	4.867	1.05%
15%	0.903	0.818	0.852	4.917	4.967	1.01%
20%	0.919	0.818	0.870	4.991	5.050	1.17%
25%	0.938	0.834	0.889	5.092	5.150	1.13%
30%	0.958	0.848	0.908	5.200	5.250	0.95%
35%	0.977	0.856	0.926	5.292	5.367	1.40%
40%	1.000	0.864	0.926	5.383	5.483	1.82%
45%	1.026	0.879	0.944	5.509	5.583	1.33%
50%	1.051	0.886	0.963	5.625	5.700	1.32%
55%	1.086	0.894	0.981	5.774	5.783	0.16%
60%	1.120	0.909	1.000	5.933	5.883	0.85%
65%	1.153	0.925	1.019	6.084	6.000	1.40%
70%	1.185	0.939	1.037	6.233	6.117	1.90%
75%	1.218	0.955	1.074	6.400	6.233	2.68%
80%	1.259	0.970	1.092	6.583	6.378	3.21%
85%	1.301	0.993	1.130	6.792	6.550	3.69%
90%	1.352	1.023	1.176	7.049	6.750	4.43%
95%	1.431	1.075	1.259	7.466	7.100	5.15%

(d)

Figure 1: Comparisons between the travel time density functions synthesized from individual segment percentiles (squares) and the observed values (dots).